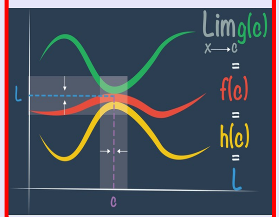
Calculus I

Lecture 8



Feb 19-8:47 AM

Suppose c is constant and

$\lim\limits_{x\to a} f(x)$ & $\lim\limits_{x\to a} g(x)$ both exist.

1) $\lim\limits_{x\to a} c = c$

2) $\lim\limits_{x\to a} c f(x) = c \lim\limits_{x\to a} f(x)$

3) $\lim\limits_{x\to a} [f(x) + g(x)] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$

4) $\lim\limits_{x\to a} [f(x) - g(x)] = \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} g(x)$

ex: Given $\lim\limits_{x\to a} f(x) = 5$, $\lim\limits_{x\to a} g(x) = -3$

Find

1) $\lim\limits_{x\to a} [2f(x) - 10] = \lim\limits_{x\to a} 2f(x) - \lim\limits_{x\to a} 10$

$= 2 \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} 10$

$= 2 \cdot 5 - 10 = \boxed{0}$

2) $\lim\limits_{x\to a} [6f(x) + 10g(x)]$

$= 6 \lim\limits_{x\to a} f(x) + 10 \lim\limits_{x\to a} g(x) = 6\cdot 5 + 10(-3) = \boxed{0}$

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5) $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

6) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$

Ex: $\lim_{x \to a} f(x) = -4$, $\lim_{x \to a} g(x) = 4$

find

1) $\lim_{x \to a} [f(x) \cdot g(x) + 16] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) + \lim_{x \to a} 16$

$= (-4)(4) + 16 = \boxed{0}$

2) $\lim_{x \to a} \frac{f(x) + 8}{g(x) - 8} = \frac{\lim_{x \to a} f(x) + 8}{\lim_{x \to a} g(x) - 8} = \frac{-4+8}{4-8} = \frac{4}{-4} = \boxed{-1}$

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7) $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$, n is positive integer.

8) $\lim_{x \to a} x = a$

9) $\lim_{x \to a} x^n = a^n$, n is Pos. integer.

10) $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ n is positive integer, $a > 0$

ex: Given $\lim_{x \to a} f(x) = -2$, find $\lim_{x \to a} [f(x)]^3$

$\lim_{x \to a} [f(x)]^3 = [\lim_{x \to a} f(x)]^3 = (-2)^3 = \boxed{-8}$

Evaluate

$\lim_{x \to 4} (2\sqrt{x} - x^2) = 2 \lim_{x \to 4} \sqrt{x} - [\lim_{x \to 4} x]^2$

$= 2 \cdot \sqrt{4} - 4^2 = 4 - 16 = \boxed{-12}$

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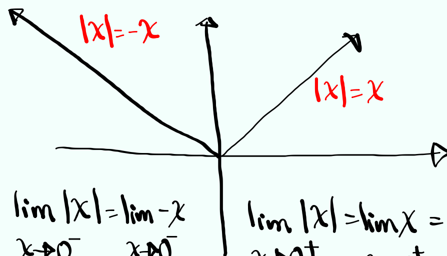
When do we know the limit exists?

$\lim_{x\to a} f(x)$ exists when $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

ex: $\lim_{x\to 0} |x| = 0$



$\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} -x = 0$

$\lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$

$\lim_{x\to a} f(x) = f(a)$ only if a is in the domain of $f(x)$

Direct Subs.
(Plug it in)

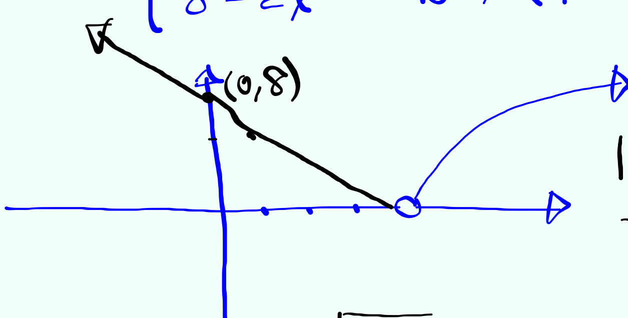
Sep 9-7:54 AM

Consider the function below

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \end{cases} \Rightarrow x \neq 4$$

Domain
$(-\infty, 4) \cup (4, \infty)$

Range
$(0, \infty)$



$\lim_{x\to 4} f(x) = 0$

$\lim_{x\to 4^+} f(x) = \sqrt{4-4} = 0$, $\lim_{x\to 4^-} f(x) = 8 - 2(4) = 0$

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Suppose $g(x) \le f(x) \le h(x)$ on (a, c)

and $\lim_{x \to b} g(x) = L = \lim_{x \to b} h(x)$ where $b \in (a, c)$

then $\lim_{x \to b} f(x) = L$ Squeeze theorem

Given $2 - x^2 \le f(x) \le \cos x + 1$ for $[-1, 1]$

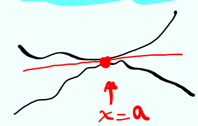
Find $\lim_{x \to 0} f(x)$

$\lim_{x \to 0} (\cos x + 1) = \cos 0 + 1 = 1 + 1 = \boxed{2}$

by S.T.,

$\lim_{x \to 0} (2 - x^2) = 2 - 0^2 = \boxed{2}$

$\lim_{x \to 0} f(x) = 2$



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Evaluate $\lim_{x \to -1} \sqrt{\dfrac{2x^2 - 1}{3 + 2x}}$

$= \sqrt{\lim_{x \to -1} \dfrac{2x^2 - 1}{3 + 2x}} = \sqrt{\dfrac{\lim_{x \to -1} (2x^2 - 1)}{\lim_{x \to -1} (3 + 2x)}}$

$= \sqrt{\dfrac{2(-1)^2 - 1}{3 + 2(-1)}} = \sqrt{\dfrac{1}{1}} = \boxed{1}$

Sep 9-8:12 AM

Given $4x-9 \le g(x) \le x^2-4x+7$ for $x \ge 0$

Find $\lim\limits_{x\to 4} g(x)$

$= \boxed{7}$ by S.T.

$\lim\limits_{x\to 4}(4x-9) = 16-9 = 7$

$\lim\limits_{x\to 4}(x^2-4x+7) = 16-16+7 = 7$

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Evaluate

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \frac{0}{0} \quad \text{I.F.}$$

$6-x-4$

$\cancel{=2-x}$

$$\lim_{x\to 2} \frac{(\sqrt{6-x}-2)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}{(\sqrt{3-x}-1)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}$$

$3-x-1$

$\cancel{=2-x}$

$$= \lim_{x\to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

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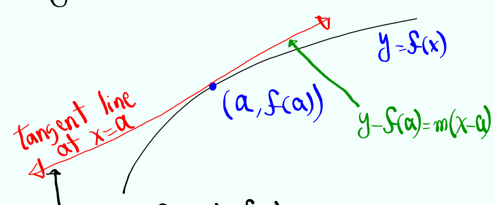
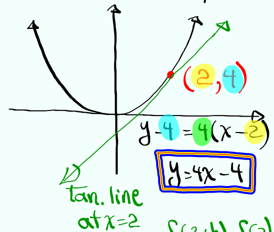
Looking ahead



$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Find eqn. of tan. line to the graph of $f(x) = x^2$ at $x=2$.



$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 - 4}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = \lim_{h\to 0}(4+h) = 4$$

$y - 4 = 4(x-2)$

$\boxed{y = 4x - 4}$

$$m = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

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